2021

STATISTICS — GENERAL

First Paper

Full Marks : 100

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group – A

(Marks : 50)

Answer ***question no.*** **1** and ***any three*** questions from ***question nos.*** **2-7**.

1. Answer ***any four*** questions : 2×4

(a) What do you mean by Statistical Data?

(b) Name the different parts of a table in connection with tabulation of data.

(c) If A.M. and C.V. of a data on a variable x are 10 and 50% respectively, find the standard deviation of $(3 - 2x)$.

(d) Define rth order raw moment.

(e) Interpret the cases $r_{xy} = -1$, $r_{xy} = +1$.

(f) If $r_{12} = 0.4$, $r_{13} = r_{23} = 0.5$, find $r_{1.23}$.

(g) Suggest a measure of skewness based on quartiles.

(h) Based on n pairs of values write down the linear regression equation of x on y.

2. (a) Distinguish, with examples, between Frequency and Non-frequency types of data.

(b) Describe the situation where Pie Chart is used. Also, describe the method of construction of Pie Chart.

(c) Describe the steps of construction of a frequency distribution of a continuous variable. 4+5+5

3. (a) Show that mean deviation about median cannot be greater than S.D.

(b) There are two sets with n_1 and n_2 values of variable x having means $\bar{x}_1$ and $\bar{x}_2$, variances s_1^2 and s_2^2 respectively. Show that if s^2 is the combined variance then

$$(n_1 + n_2)^2 s^2 = (n_1 + n_2)\left(n_1 s_1^2 + n_2 s_2^2\right) + n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2$$ 6+8

4. (a) Show that $s^2 \le R^2/4$ where s and R are the s.d. and the range of a set of n observations. Hence show that the s.d. of scores in a paper with full marks 100 cannot be greater than 50.

(b) Define scatter diagram. If x and y are uncorrelated, obtain the correlation between $x + y$ and $x - y$.

(c) What is coefficient of variation? Write its uses. (4+2)+(2+3)+(1+2)

5. (a) What do you mean by skewness of a frequency distribution? State two measures of skewness. Show that $-3 \le \dfrac{\text{mean} - \text{mode}}{\text{s.d.}} \le 3$.

(b) Obtain first four central moments in terms of raw moments. (3+2+4)+5

6. (a) Describe the principle of least squares.

(b) Interpret the cases $r = 0$ and $r = \pm 1$.

(c) Applying least squares method, determine the regression line of y on x on the basis of n pairs of values of x and y. 4+3+7

7. (a) Derive Spearman's Rank Correlation coefficient for no tie case.

(b) In a trivariate distribution show that the regression equation of x_1 on x_2 and x_3 is

$\dfrac{(x_1 - \bar{x}_1)}{s_1} R_{11} + \dfrac{(x_2 - \bar{x}_2)}{s_2} R_{12} + \dfrac{(x_3 - \bar{x}_3)}{s_3} R_{13} = 0$ where $\bar{x}_i$ and s_i are the means and standard deviations of x_i ($i = 1, 2, 3$) respectively and R_{1j} is the cofactor of r_{1j} in $|R|$, $j = 1, 2, 3$; R is the correlation matrix. 6+8

Group – B

(Marks : 50)

Answer ***question no.* 8** and ***any three*** questions from the rest.

8. Answer ***any four*** questions : 2×4

(a) Write down the sample space when a coin is tossed four times.

(b) If $P(A) = 1/2$, $P(B) = 3/5$ and $P(A \cap B) = 1/3$, find $P(A^c | B^c)$.

(c) Distinguish between an elementary event and a composite event.

(d) If X follows $N(0, 1)$, find $Var(X^2)$.

(e) Give the points of inflexion of a normal distribution with mean 20 and variance 4.

(f) Calculate $\dfrac{\text{Mean} - \text{Mode}}{\text{SD}}$ of a Poisson distribution with parameter 7/4.

(g) Find the mean of a geometric distribution with parameter p.

(h) If $V(X) = V(Y) = 1/4$ and $V(X–Y) = 1/3$, what is the correlation between X and Y?

9. (a) Derive the simplified form for $P(A \cup B \cup C)$ when the 3 events A, B and C are not mutually exclusive.

(b) The nine digits 1, 2, ..., 9 are arranged in random order to form a nine-digit number. Find the probability that 1, 2 and 3 appear as neighbours if (i) 1, 2, 3 is in natural order (ii) the order is not maintained.

(c) Distinguish between discrete and continuous random variables. 5+5+4

10. (a) Give the classical definition of probability. What are its limitations?

(b) State and prove Bayes' theorem in probability.

(c) Three persons A, B and C take turns in tossing a fair coin. He who gets the first head wins the game. Find the probability of A to win. (2+2)+(2+4)+4

11. (a) The following is the distribution function of a discrete random variable X:

x:	-3	-1	0	1	2	3	5	8
$F(x)$:	0.10	0.30	0.45	0.50	0.75	0.90	0.95	1.00

Find (i) the probability mass function of X. (ii) $P(-3 \le X \le 3)$ and $P(X \ge 3 \mid X > 0)$

(b) Distinguish between mutually independent and pairwise independent events.

(c) Define a bivariate normal distribution. Discuss any two properties of this distribution. (2+2+2)+4+4

12. (a) Obtain the recurrence relation for central moments for a binomial distribution. Hence comment on the skewness and kurtosis of the distribution.

(b) Stating clearly the assumptions show that Poisson distribution is a limiting case of binomial distribution. (6+4)+4

13. (a) If μ_{2r} denotes the $2r$th order central moment of a $N(\mu, \sigma^2)$ distribution, show that

$$\mu_{2r} = \sigma^{2r} (2r-1)(2r-3)...3.1.$$

Hence comment on the kurtosis of the distribution.

(b) Let X be a random variable with p.d.f. $f(x) = kx^4(1-x)^5$, $k > 0$, $0 < x < 1$. Find the constant k. Also find the mean and variance of X. (6+2)+(2+2+2)

14. (a) Write the statement of central limit theorem for I.I.D. random variables.

(b) State and prove the weak law of large numbers (WLLN).

(c) Test whether WLLN holds or not for the sequence of independent random variables $\{X_n\}, n = 1, 2, ...$ such that

$$P(X_n = n) = P(X_n = -n) = \frac{1}{2\sqrt{n}}$$

$$P(X_n = 0) = 1 - \frac{1}{\sqrt{n}}$$

2+6+6